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COMMENT

Comment on 'Focusing light using negative refraction'

R Ruppin[1]

The Blackett Laboratory, Imperial College, London SW7 2AZ, UK

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Abstract
In a recent paper, Pendry and Ramakrishna (2003 *J. Phys: Condens. Matter* **15** 6345) have introduced the concept of optically complementary media. Here it is shown that the focusing ability of such media follows directly from the symmetry of Maxwell's equations.

Pendry and Ramakrishna [1] have shown that a slab with $\varepsilon = \mu = -1$, which optically cancels an equal thickness of space with $\varepsilon = \mu = 1$, is a special case of a wider class of focusing geometries. Whenever the permittivity ε and the permeability μ are antisymmetric functions about a plane, a similar cancellation effect exists. The proof presented in [1] was based on decomposing the electric and magnetic fields into Fourier components, and analysing the behaviour of these as dictated by Maxwell's equations. Here we state the complementary media theorem in its most general form and demonstrate that it follows directly from the symmetry properties of Maxwell's equations.

Complementary media theorem. *Let the permittivity and the permeability be* ε_1 *and* μ_1 *in region 1, defined by* $-a < z < 0$*, and* ε_2 *and* μ_2 *in region 2, defined by* $0 < z < a$ *(figure 1). Let these functions be antisymmetric about the* $z = 0$ *plane:*

$$\varepsilon_2(x, y, z) = -\varepsilon_1(x, y, -z) \quad (1)$$

$$\mu_2(x, y, z) = -\mu_1(x, y, -z). \quad (2)$$

If

$$\vec{E}_1(x, y, z) = (E_{1x}(x, y, z), E_{1y}(x, y, z), E_{1z}(x, y, z)) \quad (3)$$

$$\vec{H}_1(x, y, z) = (H_{1x}(x, y, z), H_{1y}(x, y, z), H_{1z}(x, y, z)) \quad (4)$$

is a solution of Maxwell's equations in region 1, then

$$\vec{E}_2(x, y, z) = (E_{1x}(x, y, -z), E_{1y}(x, y, -z), -E_{1z}(x, y, -z)) \quad (5)$$

$$\vec{H}_2(x, y, z) = (H_{1x}(x, y, -z), H_{1y}(x, y, -z), -H_{1z}(x, y, -z)) \quad (6)$$

[1] On leave from: Soreq NRC, Yavne 81800, Israel.

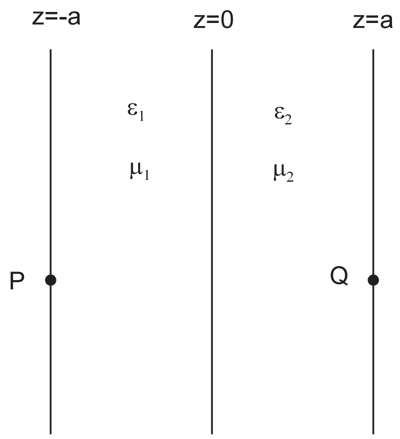

Figure 1. Pair of complementary media.

is the solution in the region 2, which satisfies the Maxwell boundary conditions at the interface $z = 0$*. Thus, the x and y components of the fields are symmetric functions of z, whereas the z components are antisymmetric functions of z.*

Proof. Consider the Maxwell equation satisfied by the fields in region 1:

$$\nabla \times \vec{E}_1 = \mathrm{i}\omega\mu_0\mu_1\vec{H}_1. \tag{7}$$

Taking the x component

$$\frac{\partial E_{1z}}{\partial y} - \frac{\partial E_{1y}}{\partial z} = \mathrm{i}\omega\mu_0\mu_1 H_{1x} \tag{8}$$

and using (2), (5) and (6) we obtain

$$\frac{\partial E_{2z}}{\partial y} - \frac{\partial E_{2y}}{\partial z} = \mathrm{i}\omega\mu_0\mu_2 H_{2x}. \tag{9}$$

Here we have also used the fact that, since E_y is symmetric with respect to z, its derivative with respect to z is antisymmetric. Taking the z component of (7)

$$\frac{\partial E_{1y}}{\partial x} - \frac{\partial E_{1x}}{\partial y} = \mathrm{i}\omega\mu_0\mu_1 H_{1z} \tag{10}$$

and using (2), (5) and (6) we obtain

$$\frac{\partial E_{2y}}{\partial x} - \frac{\partial E_{2x}}{\partial y} = \mathrm{i}\omega\mu_0\mu_2 H_{2z}. \tag{11}$$

From (9) (and a similar equation for the y component H_{2y}) and (11) we see that the fields in region 2 satisfy the Maxwell equation

$$\nabla \times \vec{E}_2 = \mathrm{i}\omega\mu_0\mu_2\vec{H}_2. \tag{12}$$

The symmetry properties of the Maxwell equation $\nabla \times \vec{H} = -\mathrm{i}\omega\varepsilon_0\varepsilon\vec{E}$ are completely analogous. Thus, the fields defined by (5) and (6) satisfy the Maxwell equations in region 2. From (5) and (6) it is also seen that the Maxwell boundary conditions at $z = 0$, of the continuity of the tangential components E_x, E_y, H_x, H_y and of the normal displacement and induction components εE_z and μH_z, are automatically satisfied. Finally, we note that the fields just to the left of any point P on the $z = -a$ interface (figure 1) are identical to the fields just to the right of the corresponding point Q on the $z = a$ interface, and hence the complementary media can be eliminated from the optical response calculation, as discussed in [1]. □

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References

[1] Pendry J B and Ramakrishna S A 2003 *J. Phys.: Condens. Matter* **15** 6345